Heun's method

1. Use four steps of Heun's method to approximate a solution on the interval [0, 1] to the initial-value problem defined by

$$y^{(1)}(t) = -y(t) + t - 2$$
$$y(0) = 1$$

Answer: 1.0, 0.375, –0.05859375, –0.3426513672, –0.5098838806

2. Use eight steps of Heun's method to approximate a solution on the interval [0, 1] to the initial-value problem defined that shown in Question 1.

Answer: To ten digits of significance, 1, 0.65625, 0.3674316406, 0.1271076202, -0.07040499151, -0.2301231566, -0.3564759117, -0.4533732658, -0.5242670237.

3. If the actual solution is $y(t) = 4e^{-t} + t - 3$, argue that this method is indeed $O(h^3)$ for a single step.

Answer: To ten significant digits, the error of the approximation of $y(0.25)$ in Question 1 is 0.009797 and the error of the approximation of $y(0.125)$ in Question 2 is 0.001225, and this second value is approximately one eighth the error of the first.

4. If the actual solution is $y(t) = 4e^{-t} + t - 3$, argue that this method is indeed $O(h^2)$ over multiple steps.

Answer: $y(1) = 4e^{-1} + 1 - 3 \approx -0.5284822353142307136$, so the error of the approximation in Question 1 is approximately 0.01860 while the error with the second approximation is 0.004215, which is approximately one quarter that of the previous approximation.